Joshua J. Friess, Tao Han, Dan Hooper
*Department of Physics, University of Wisconsin, 1150 University Avenue, Madison, WI 53706*
(April, 2002)

We construct the open-string scattering amplitudes for neutrino-gluon collisions and evaluate the high energy neutrino-nucleon scattering cross section via string state excitations in the TeV string-scale scenario. We find that the neutrino-gluon scattering is the dominant contribution, $5-10$ times larger than neutrino-quark processes, though black hole production may be larger than the string contribution at higher energies. We illustrate the observability of the string signal at the Auger Observatory and the IceCube neutrino telescope for the string scale $M_S \simeq 1$ TeV.

04.50.+h, 04.60.-m, 95.55.Vj, 95.85.Ry

*Introduction:* Theories with a low fundamental string scale ($M_S$) [1] may help explain the apparent mass hierarchy between the electroweak scale and the Planck scale ($M_{\rm Pl}$) [2,3]. These interesting new scenarios may also lead to rich low energy phenomenology [4] that may be observable in near-future high energy experiments [5]. Generally speaking, at an energy scale probing a distance smaller than the size of extra dimensions, one expects to explore the nature of higher dimensional physics, where gravitons propagating in the bulk play an important role in the dynamics [2]. At energies much larger than the fundamental scale $M_S$, black hole (bh) production with semi-classical treatments may be the dominant phenomenon [6,7]. At energies near the string scale threshold, however, the physics may be more involved, and the string resonance states should become very important [8]. Perturbative arguments show that the (open) string scattering amplitudes should be dominant over the (closed string) graviton processes.

In this paper, we study the TeV string state excitations via high energy cosmic neutrinos. The motivation is multi-fold. First, high energy cosmic rays provide a natural accelerator. Neutrinos are not subject to the GZK cutoff and may arrive at the Earth with ultra-high energies, leading to potentially spectacular events [9] in the atmosphere or in the Earth's crust. Second, models with a low string scale typically predict enhanced neutrino interactions with matter at higher energies [10,11]. For instance, an effective field theory calculation for large extra dimension scenarios leads to a $\nu g$ scattering amplitude $\mathcal{M} \sim s^2/M_S^4$ [11], where $s$ is the c.m. energy squared. This energy growth, however, violates partial wave unitarity near the string scale, thus invalidating the perturbative treatment. On the other hand, the open-string scattering amplitudes, which are manifestly unitary, should be the appropriate description for physics in this energy regime [8,12]. In fact, the neutrino-nucleon scattering cross section has been constructed in this scenario [13], and the physical consequences have been studied in the context of the km-squared neutrino telescope [14]. However, the scattering amplitudes were constructed only for neutrino-quark interactions. Since we are exploring the new physics typically at energies of a TeV, one expects that gluons in a nucleon would become the dominant partons to participate the interaction. Finally, it is important to explore the relative contributions from string excitations and black hole production. In the next section, we construct the open-string amplitudes for neutrino-gluon $\nu g$ scattering. We then study the string resonances in the $s$-channel and evaluate the neutrino-nucleon ($\nu N$) scattering cross section. We compare the string state contribution with that of black hole production, and finally, we demonstrate the signal observability at the Pierre Auger air-shower array observatory, as well as the IceCube neutrino telescope.

*$\nu_L g$ Elastic Scattering Amplitude:* The general tree-level open-string amplitudes can be expressed by [15]

$$\mathcal{M}(1,2,3,4) = g^2[A_{1234}\cdot S(s,t)\cdot T_{1234} + A_{1324}\cdot S(t,u)\cdot T_{1324} + A_{1243}\cdot S(s,u)\cdot T_{1243}\,] , \quad (1)$$

where $(1,2,3,4)$ denote the external states with momenta directed inward. $A_{ijkl}$ are the kinematic factors for the color-ordered amplitudes [16], associated with the expansion basis $T_{ijkl}$, the so-called Chan-Paton factors. $S(s,t)$ is essentially the Veneziano amplitude

$$S(s,t) = \frac{\Gamma(1-\alpha' s)\Gamma(1-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)} , \quad (2)$$

where $\alpha' = 1/{M_S}^2$ is the Regge slope. In the zero-slope limit $\alpha' s \to 0$, the Veneziano amplitudes approach unity, and Eq. (1) should reduce to the field theory result at energies far below the string scale.

Identifying $(\nu_L, g, g, \nu_L) \to (1,2,3,4)$, we can easily derive the relevant factors for our amplitudes. For instance, for a right-handed gluon, we have [12,16]

$$A_{1234} = -4\frac{u}{t}\sqrt{\frac{u}{s}},\ A_{1324} = -4\frac{s}{t}\sqrt{\frac{u}{s}},\ A_{1243} = -4\sqrt{\frac{u}{s}}\ .$$

Substituting these back into Eq. (1) and taking the (3,4) momenta outgoing, we get the full amplitude:

$$\mathcal{M}(\nu_L g_R \to \nu_L g_R) = -4g^2\frac{1}{t}\sqrt{\frac{-u}{s}} \times [uS(s,t)T_{1234} + sS(t,u)T_{1324} + tS(s,u)T_{1243}]\ . \quad (3)$$

For a model given by embedding the SM fields into a D-brane structure in string theories, one will be able to calculate explicitly the Chan-Paton $T$ factors. For a typical embedding into a $U(N)$ gauge group (with the group generators normalized to $Tr(t^a t^{b\dagger}) = \frac{1}{2}\delta^{ab}$), the Chan-Paton factors can be $T \sim 1/4$ to 1. For instance, in a QED toy model [12], the Chan-Paton factors for $e^-_L e^+_R \to \gamma_L \gamma_R$ are $T = \pm 1/4$. As a consistency check of this method, Eq. (3) leads to the familiar QED Compton scattering amplitude when the Chan-Paton factors are evaluated to be $T_{1234} = T_{1324} = -T_{1243} = 1/4$, following [12].

On the other hand, explicit constructions of string embeddings often lead to new states that are incompatible with the SM at low energies. Instead, we take a rather model-independent approach: although we do not assign an explicit representation of the $U(N)$ generators to the SM particles, we can parameterize the $T$'s by requiring that Eq. (3) reproduce the SM result at low energies. Since the tree-level SM result for the $\nu g$ scattering vanishes and the Veneziano amplitudes approach unity at low energies ($\alpha' s \to 0$), we require

$$0 = u\ T_{1234} + s\ T_{1324} + t\ T_{1243}\ . \tag{4}$$

This relation is satisfied in the massless limit if and only if $T_{1234} = T_{1324} = T_{1243} \equiv T$. The corresponding amplitude for a left-handed gluon can be obtained in a similar way. We thus arrive at our final expressions:

$$\mathcal{M}(\nu_L g_R \to \nu_L g_R) = -4g^2 T\ \frac{1}{t}\ \sqrt{\frac{-u}{s}} \times \left[uS(s,t) + sS(t,u) + tS(s,u)\right], \tag{5}$$

$$\mathcal{M}(\nu_L g_L \to \nu_L g_L) = \mathcal{M}(\nu_L g_R \to \nu_L g_R)_{u \leftrightarrow s}\ . \tag{6}$$

Since we are uninterested in the particular color structure in practice, and for simplicity of the presentation, we have taken the Chan-Paton factors for different gluons to be equal. The scattering amplitude is completely specified by the string coupling $g$, string scale $M_S$, and a model parameter $T$.

*String Resonances and the $\nu N$ Cross Section:* It is informative to understand the physical interpretation of the $\nu g$ amplitude constructed above. The Veneziano amplitude of Eq. (2) presents $s$-channel poles, as well as $t$- and $u$-channel exchanges. Obviously, the $s$- and $u$-channels are due to exchanges of fermionic states of $(\mathbf{8},\mathbf{2})$ under $SU_C(3)\otimes SU_L(2)$, called lepto-gluons ($\nu_8$). The $t$-channel involves new exotic bosons which may be in the adjoint representations of the SM gauge groups.

Although our amplitudes of Eqs. (5–6) reduce to the SM results at $s \ll M_S^2$, at energies near or above the string threshold, the string resonances in the $s$-channel dominate. The Veneziano amplitude has simple poles at $s = nM_S^2$ for any positive integer $n$,

$$S(s,t) \approx M_S^2\ \sum_{n=1}^{\infty} \frac{(t/M_S^2)(t/M_S^2+1)\cdots(t/M_S^2+n-1)}{(n-1)!(s-nM_S^2)}\ .$$

Off-resonance contributions to the cross section are negligible. There are also terms proportional to $S(t,u)$. We will neglect these terms and approximate the amplitude near its $s$-channel poles [13]. The amplitude for right-handed gluons now becomes

$$\mathcal{M}(\nu_L g_R \to \nu_8 \to \nu_L g_R) \approx \sum_{n=1}^{\infty} A_n\ , \tag{7}$$

where

$$\begin{aligned} A_n &= \frac{-4g^2 T M_S^2}{(n-1)!(s-nM_S^2)}\ \frac{1}{t}\ \sqrt{\frac{-u}{s}} \\ &\times \left[u\frac{t}{M_S^2}(\frac{t}{M_S^2}+1)\cdots(\frac{t}{M_S^2}+n-1) + (t \leftrightarrow u)\right] \\ &= \frac{-8g^2 T M_S^2}{(n-1)!(s-nM_S^2)}\ \frac{u}{t}\ \sqrt{\frac{-u}{s}} \\ &\times \frac{t}{M_S^2}(\frac{t}{M_S^2}+1)\cdots(\frac{t}{M_S^2}+n-1) \quad \text{for odd } n \end{aligned} \tag{8}$$

and $A_n = 0$ for even $n$. It is convenient to expand the amplitude [13] in terms of the Wigner functions $d^J_{mm'}$. Since both initial and final states for $\nu_L g_R \to \nu_L g_R$ have total helicity $J_Z = 3/2$, the result reads

$$A_n = \frac{8g^2 T n M_S^2}{s - nM_S^2} \sum_{J=3/2}^{n+1/2} \alpha_n^J d^J_{3/2,3/2}\ , \tag{9}$$

where the coefficient $\alpha_n^J$ satisfies the normalization relation $\sum_{J=3/2}^{n+1/2} \left|\alpha_n^J\right| = 1$.

Equipped with the amplitude for $\nu_L g_R \to (\nu_8)_n^J \to \nu_L g_R$, we can determine the partial decay width for $(\nu_8)_n^J \to \nu_L g_R$:

$$\Gamma_n^J \equiv \Gamma((\nu_8)_n^J \to \nu_L g_R) = \frac{g^2}{2\pi}\frac{|T|}{2J+1}\sqrt{n}M_S\left|\alpha_n^J\right|\ . \tag{10}$$

In the narrow-width approximation, this leads to the cross section $\sigma(\nu_L g_R \to (\nu_8)_n^J)$

$$\begin{aligned} \sigma_n^J(\nu_L g_R) &= \frac{4\pi^2 \Gamma_n^J}{\sqrt{n}M_S}(2J+1)\delta(s-nM_S^2) \\ &= 2\pi g^2 \left|T\alpha_n^J\right|\delta(s-nM_S^2)\ . \end{aligned} \tag{11}$$

We now sum these partial cross sections for a fixed odd $n$, obtaining

$$\sigma_n \equiv \sum_J \sigma_n^J(\nu_L g_R) = \tilde{\sigma}_n \delta(1 - nM_S^2/s)\ , \tag{12}$$

where $\tilde{\sigma}_n \equiv 2\pi g^2 |T|/nM_S^2$. The partial cross sections for left-handed gluons turn out to be identical to Eq. (12).

Finally, the neutrino-nucleon deeply inelastic scattering cross section is given by

$$\sigma(\nu_L N) = \sum_{n=1}^{n_{cut}} \sum_f \tilde{\sigma}_n(\nu_L f)\ xf(x,Q^2)\ , \tag{13}$$

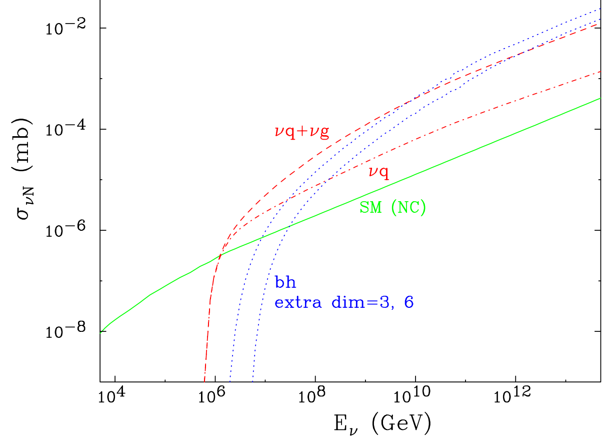


FIG. 1. $\nu N$ cross sections via TeV string resonances from $\nu q + \nu g$ contribution (the dashed curve) and $\nu q$ only (the dotdashed). The string scale is taken to be $M_S$=1 TeV and the Chan-Paton factor is $T = 1/2$. Also plotted are the SM neutral current prediction (solid curve) and the black hole production (dotted) with 3 (upper) and 6 extra dimensions.

where $x = nM_S^2/S$ is the energy fraction carried by a parton; $S$ is the neutrino-nucleon c.m. energy squared, and $Q^2$ is taken to be $nM_S^2$. The sum $f$ is over all contributing partons in the nucleon. Quark parton contributions were evaluated in [13]. As part of our current motivation, we expect that the gluonic contribution as constructed above will dominate at high energies.

We present our results in Fig. 1. The solid curve shows the prediction for the SM neutral current process. We have used the CTEQ5-DIS parton distributions, extended to $x < 10^{-5}$ using the methods in [17]. The neutrino-nucleon cross sections via string excitations with and without the gluon contribution are depicted by the dashed and dotdashed curves, respectively. Throughout our presentation, we have taken the string coupling $g = 1$, which keeps the effective coupling $g^2/4\pi$ perturbative. For comparison, we have taken $T = 1/2$, which puts the Chan-Paton trace factor for the gluons to be the same as that for the quarks when $a = b = 5$ in the parameterization of [13]. We have only summed over the resonances to $n_{cut} = 50$. Our results are rather insensitive to the choice of $n_{cut}$. For example, taking $n_{cut} = 20$ (80) instead reduces (increases) our final results by about 10%. The new physics threshold starts near $E_\nu \sim 10^3$ TeV, which corresponds to our choice of $M_S = 1$ TeV. Due to the large gluon luminosity at high energies, including the gluons in the total cross section increases the final result by about a factor of $5 - 10$ for $E_\nu \sim 10^5 - 10^{10}$ TeV.

As commented in the introduction, graviton exchange in the large extra dimension scenario is the leading effect below $M_S$ [4,14]. However, black hole production far above $M_S$ may be dominant [6,7]. We show the bh production with the geometric cross section [7] for $s \geq M_S = 1$ TeV by the dotted curves with 3 (upper) and 6 large extra dimensions [18]. Indeed, the bh cross section may take over for $E_\nu \gtrsim 10^6$ TeV. We finally note that the cross section of the order $\mu$b at $E_\nu \sim 10^{20}$ eV is much too low to account for the ultra-high energy cosmic ray events that violate the GZK bound [9].

*Signatures in Air-shower Cosmic Ray Experiments and in Neutrino Telescopes:* At energies inaccessible to colliders, astroparticle physics experiments [19,20] may provide a window into new physics. Enhanced neutrino-nucleon cross sections may be an observable signature for theories with a low string scale [10,11,13,14,21].

Very high energy neutrinos are predicted to be generated in a variety of astrophysical sources. We consider two of these possibilities: neutrinos from compact sources and cosmogenic neutrinos. For the flux from compact sources, we use the limit placed by Waxman and Bahcall of $E_\nu^2 dN_\nu/E_\nu = 2 \times 10^{-8}$ GeV cm$^{-2}$s$^{-1}$sr$^{-1}$ [22]. This flux describes neutrinos from sources such as gamma-ray bursts and blazars which are assumed to also generate the highest energy cosmic rays. The cosmogenic neutrino flux is generated by cosmic rays scattering off of the cosmic microwave background [23]. We use the flux as calculated by Seckel *et al.* [24]. This flux peaks at $E_\nu \sim 10^9$ GeV and is smaller than the Waxman-Bahcall limit over all, but is also a more robust prediction. We consider two classes of next generation high energy astroparticle physics experiments: the Auger air-shower array [25] and the kilometer-scale neutrino telescope IceCube [26].

At EeV energies, neutrinos can generate atmospheric air showers observable in cosmic ray experiments. The event rates at these extremely high energies scale as the cross section of interacting neutrinos and thus can be sensitive to new physics that enhances the cross section [19]. We present our numerical results in Table I for the quasi-horizontal events with a zenith angle larger than 75°, including the prediction from the SM interactions and those from the string resonances. It is important to note that the two neutrino fluxes considered only lead to a difference of a factor of $2-3$ at these very high energies, making the uncertainties due to the unknown flux less severe. With a few years of data taking, it is conceivable to establish a signal for $M_S \sim 1$ TeV statistically over the SM expectation at the Auger Observatory.

Neutrino telescopes, optimized for TeV-PeV neutrinos, can measure neutrino-nucleon cross sections given a sufficiently high energy neutrino flux. This is done by comparing the energy and angular distributions of showers generated in the detector to SM predictions. As cross sections increase at higher energies, the Earth becomes opaque to neutrinos, thus suppressing the up-going neutrino event rate. Down-going rates, however, become further enhanced. The ratio of up-going to down-going events is an effective measurement for the cross section at a given energy [20]. At IceCube, we consider a shower threshold energy of $E_{\rm sh}^{\rm th} = 250$ TeV (corresponding to a neutrino energy near 1 PeV) to enhance the signal to background. We see again that with a few years of data taking, a signal above the SM expectation may be identifiable at IceCube. Unlike for very high energy air-shower experiments, the two fluxes considered lead to substantially different event rates, due to the fact that the Waxman-Bahcall flux is much larger at these lower energies. We can thus see the complementarity of the Auger Observatory and IceCube.

| Auger (events/yr) | WB Flux | | Cosmogenic Flux | |
|---|---|---|---|---|
| SM ($E_{\rm sh}^{\rm th} = 10$ PeV) | 0.66 | | 0.20 | |
| $M_S = 1$ TeV | 3.15 | | 1.25 | |
| $M_S = 2$ TeV | 0.96 | | 0.34 | |
| IceCube (events/yr) | WB Flux | | Cosmogenic Flux | |
| | Down | Up | Down | Up |
| SM ($E_{\rm sh}^{\rm th} = 250$ TeV) | 8.4 | 1.8 | 0.15 | 0.012 |
| $M_S = 1$ TeV | 14.8 | 2.2 | 0.72 | 0.024 |
| $M_S = 2$ TeV | 8.7 | 1.9 | 0.22 | 0.016 |

TABLE I. The SM and string excitation event rates for ultra-high energy quasi-horizontal air showers in the Auger Observatory, and down-going and up-going showers (in $2\pi$ sr) in IceCube. The Waxman-Bahcall [22] and cosmogenic [24] fluxes are considered. The Chan-Paton factor is taken as 1/2.

*Summary:* We constructed the neutrino-gluon amplitudes as open-string scattering, and subsequently studied neutrino-nucleon collisions via string state excitations in the low string scale scenario. We found that neutrino-gluon scattering is the dominant process, $5-10$ times larger in rate than the neutrino-quark induced processes [13]. We also demonstrated that black hole production may be larger than the string state contribution at higher energies. Finally, we evaluated the signal rates at the Auger Observatory and the IceCube neutrino telescope and illustrated the possibility of observing the signal if the string scale is near 1 TeV.

*Acknowledgments*: We thank Jonathan Feng, Maxim Perelstein, and Hallsie Reno for helpful discussions. This work was supported in part by a DOE grant No. DE-FG02-95ER40896 and in part by the Wisconsin Alumni Research Foundation.


[1] I. Antoniadis, Phys. Lett. **B246**, 377 (1990); J.D. Lykken, Phys. Rev. **D54**, 3693 (1996).

[2] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys. Lett. **B429**, 263 (1998); I. Antoniadis *et al.*, Phys. Lett. **B436**, 257 (1998).

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[4] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, Phys. Rev. **D59**, 086004 (1999).

[5] G. Guidice, R. Rattazzi and J. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J.D. Lykken and R. Zhang, Phys. Rev. **D59**, 105006 (1999); E.A. Mirabelli, M. Perelstein and M.E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J. Hewett, Phys. Rev. Lett. **82**, 4765 (1999); T. Rizzo, Phys. Rev. **D59**, 115010 (1999); H. Davoudiasl, J.L. Hewett and T.G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000).

[6] P.C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. **B441**, 96 (1998); T. Banks and W. Fischler, `hep-th/9906038`; R. Emparan, G.T. Horowitz and R.C. Myers, Phys. Rev. Lett. **85**, 499 (2000);

[7] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87**, 161602 (2001); S.B. Giddings and S. Thomas, Phys. Rev. **D65**, 056010 (2002).

[8] G. Shiu and S.H. Tye, Phys. Rev. **D58**, 106007 (1998); L.E. Ibanez, R. Rabadan and A.M. Uranga, Nucl. Phys. **B542**, 112 (1999); I. Antoniadis, C. Bachas and E. Dudas, Nucl. Phys. **B560**, 93 (1999); K. Benakli, Phys. Rev. **D60**, 104002 (1999); E. Accomando, I. Antoniadis and K. Benakli, Nucl. Phys. **B579**, 3 (2000).

[9] For recent reviews, see *e. g.*, R. Gandhi, Nucl. Phys. Proc. Suppl. **91**, 453 (2000); T.J. Weiler, `hep-ph/0103023`.

[10] S. Nussinov and R. Shrock, Phys. Rev. **D59**, 105002 (1999); G. Domokos and S. Kovesi-Domokos, Phys. Rev. Lett. **82**, 1366 (1999).

[11] P. Jain, D.W. McKay, S. Panda and J.P. Ralston, Phys. Lett. **B484**, 267 (2000).

[12] S. Cullen, M. Perelstein and M. E. Peskin, Phys. Rev. **D62**, 055012 (2000).

[13] F. Cornet, J. I. Illana and M. Masip, Phys. Rev. Lett. **86**, 4235 (2001).

[14] J. Alvarez-Muniz *et al.*, Phys. Rev. Lett. **88**, 021301 (2002).

[15] M.R. Garousi and R.C. Myers, Nucl. Phys. **B475**, 193 (1996); A. Hashimoto and I.R. Klebanov, Nucl. Phys. Proc. Suppl. **55B**, 118 (1997).

[16] M.L. Mangano and S.J. Parke, Phys. Rep. **200**, 301 (1991).

[17] R. Gandhi *et al.*, Astropart. Phys. **5**, 81 (1996).

[18] We have adopted a convention to relate our string scale and $(4+n)$-dimensional gravity scale as $M_S = [8\pi/(2\pi)^{2n}]^{1/n+2} M_D$.

[19] A. Kusenko, `hep-ph/0203002`.

[20] D. Hooper, `hep-ph/0203239`.

[21] H. Davoudiasl, J.L. Hewett and T. G. Rizzo, hep-ph/0010066; C. Tyler, A. Olinto and G. Sigl, Phys. Rev. **D63**, 055001 (2001); J.L. Feng and A.D. Shapere, Phys. Rev. Lett. **88**, 021303 (2002); L. Anchordoqui and H. Goldberg, Phys. Rev. **D65**, 047502 (2002); R. Emparan, M. Masip and R. Rattazzi, Phys. Rev. **D65**, 064023 (2002); L. Anchordoqui *et al.*, `hep-ph/0112247`; J. Alvarez-Muniz *et al.*, `hep-ph/0202081`.

[22] E. Waxman and J.N. Bahcall, Phys. Rev. **D59**, 023002 (1999). J. N. Bahcall and E. Waxman, Phys. Rev. **D64**, 023002 (2001).

[23] F.W. Stecker, Astrophys. J. **228**, 919 (1979).

[24] R. Engel, D. Seckel and T. Stanev, Phys. Rev. D **64**, 093010 (2001).

[25] I. Allekote *et. al.* (Auger Collaboration), in *Proc. 27th International Cosmic Ray Conference*, Hamburg, Germany, 2001, p. 370; http://www.auger.org/admin/.

[26] E. Andres *et al.*, Nature **410**, 441 (2001).